Spring Progress Check Revision Material

Year 10 Set 1 - 3

Higher

Test Date: Wednesday 24 Jan

How to revise for Maths?

- *Practise is key! Attached you will find some questions to help you do that.*
- *Once you've answered the questions – mark your work.*
- *If you get something wrong, look back on what you did and try work out where your mistake is. Unsure? Take your answers to your teacher or to Maths club on a Thursday and get help ahead of the test!*
- *Good luck!*

1 Knowledge check

- When there are m ways of doing one task and n ways of doing a different task, the total number of ways the two tasks can be done is $m \times n$. *Mastery lesson 1.1*
- You can round numbers to 1 or 2 significant figures to estimate the answers to calculations, including calculations with powers and roots. *Mastery lesson 1.2*
- You can use a **prime factor tree** to write a number as the product of its **prime factors**. *Mastery lesson 1.3*
- You can use a **Venn diagram** of prime factors to work out the **highest common factor** and **lowest common multiple** of two numbers. *Mastery lesson 1.3*
- The **prime factor decomposition** of a number is the number written as the product of its prime factors. It is usually written in index form. *Mastery lesson 1.3*
- When multiplying powers, add the indices: $x^m \times x^n = x^{m+n}$
  When dividing powers, subtract the indices: $x^m \div x^n = x^{m-n}$
  To raise a power to another power, multiply the indices.
  $x^{-n} = \frac{1}{x^n}$ $x^{\frac{1}{n}} = \sqrt[n]{x}$ $x^{\frac{m}{n}} = (\sqrt[n]{x})^m$ *Mastery lesson 1.4, 1.5*
- A number in **standard form** is written in the format $A \times 10^n$, where A is a number between 1 and 10 and n is an integer. *Mastery lesson 1.6*
- To write a number in standard form:
  - work out the value of A
  - work out how many times A must be multiplied or divided by 10. This is the value of n. *Mastery lesson 1.6*
- To simplify a **surd**, identify any factors that are square numbers. *Mastery lesson 1.7*
- To **rationalise a denominator**, multiply the numerator and the denominator by the surd in the denominator and simplify. *Mastery lesson 1.7*

2 Knowledge check

- $x^m \times x^n = x^{m+n}$ $\quad x^m \div x^n = x^{m-n}$ $\quad (x^m)^n = x^{mn}$
  $x^0 = 1$ $\quad x^{-m} = \frac{1}{x^m}$ $\quad x^{\frac{1}{n}} = \sqrt[n]{x}$ ..... *Mastery lesson 2.1*
- When the two sides of a relation such as $2(x + 5) = 2x + 10$ are equal for all values of x it is called an **identity** and we write $2(x + 5) \equiv 2x + 10$ using the '$\equiv$' symbol. ..... *Mastery lesson 2.2*
- An **equation**, such as $2x = 6$, is only true for certain values of x (in this case $x = 3$). ..... *Mastery lesson 2.2*
- To expand a bracket, multiply each term inside the brackets by the term outside the brackets. $x(y + z) \equiv xy + xz$ ..... *Mastery lesson 2.2*
- Unless a question asks for a decimal answer, give non-integer solutions to an equation as exact fractions. ..... *Mastery lesson 2.3*
- To solve an equation involving fractions, multiply each term on both sides by the LCM of the denominator. ..... *Mastery lesson 2.3*

- An **expression** contains letter and number terms but no equals sign, e.g. $2ab$, $2ab + 3a^2b$, $2ab - 7$ ..... *Mastery lesson 2.4*
- An **equation** has an equals sign, terms in one letter and numbers, e.g. $2x - 4 = 9x + 1$
  You can solve it to find the value of the letter. ..... *Mastery lesson 2.4*
- An **identity** has an equals sign and is true for all values of the letters, e.g. $\frac{4x}{2} = 2x$, $x(x + y) \equiv x^2 + xy$ ..... *Mastery lesson 2.4*
- A **formula** has an equals sign and letters to represent different quantities, e.g. $A = \pi r^2$
  The letters are **variables** as their values can vary. ..... *Mastery lesson 2.4*
- The **subject** of a formula is the letter on its own, on one side of the equals sign. ..... *Mastery lesson 2.4*
- In an **arithmetic sequence** the terms increase (or decrease) by a fixed number called the **common difference**. ..... *Mastery lesson 2.5*
- When an arithmetic sequence with common difference d is input into this function machine, the output sequence has common difference $p \times d$. ..... *Mastery lesson 2.5*

- In a Fibonacci-like sequence the next number is found by adding the previous two numbers together. ..... *Mastery lesson 2.6*
- In a **geometric sequence** the terms increase (or decrease) by a **constant multiplier**. The nth term is ar^n. ..... *Mastery lesson 2.6*
- A **quadratic sequence** has n^2 and no higher power of n in its nth term. *Mastery lesson 2.6*
- The second differences of a quadratic sequence, $u_n = an^2 + bn + c$ are constant and equal to $2a$. ..... *Mastery lesson 2.6*

- The nth term of a quadratic sequence can be worked out in three steps.
  Step 1 Work out the second differences.
  Step 2 Halve the second difference to get the an^2 term.
  Step 3 Subtract the sequence an^2. You may need to add a constant, or find the nth term of the remaining terms. *Mastery lesson 2.6*
- To expand **double brackets**, multiply each term in one bracket by each term in the other bracket. *Mastery lesson 2.7*
- To **square** a single bracket, multiply it by itself, then expand and simplify, e.g. $(x + 1)^2 = (x + 1)(x + 1) = x^2 + 2x + 1$ *Mastery lesson 2.7*
- A **quadratic expression** has a squared term (and no higher power), e.g. $x^2 + 8x + 10$ *Mastery lesson 2.7*

3 Knowledge check

- A **back-to-back stem and leaf diagram** compares two sets of results. On the left-hand side the numbers are read backwards. *Mastery lesson 3.1*
- A **frequency polygon** is a graph made by joining the midpoints of the tops of the bars in a bar chart with straight lines. *Mastery lesson 3.1*
- A quicker way of drawing a frequency polygon is to plot the frequency against midpoints of each group. *Mastery lesson 3.2*
- The **modal class** (or modal group) has the highest frequency. *Mastery lesson 3.5*
- To estimate a mean from a grouped frequency table, add together the products of class midpoints and their frequencies, and divide by the total frequency. *Mastery lesson 3.5*
- If the total frequency in a grouped frequency table is n, then the median lies in the group containing the $\frac{n+1}{2}$th item of data. *Mastery lesson 3.5*
- A **time series** graph is a line graph with time plotted on the horizontal axis. *Mastery lesson 3.3*
- **Bivariate data** is data that has two variables. Points can be plotted on a **scatter diagram** to see if there is a link between them. *Mastery lesson 3.4*
- Data displays positive correlation if the points on a scatter diagram lie close to an upward-sloping straight line. Data displays negative correlation if the points on a scatter diagram lie close to a downward-sloping straight line. *Mastery lesson 3.3*
- **A line of best fit** is the line that passes as close as possible to the points on a scatter graph. *Mastery lesson 3.4*
- Using a line of best fit to predict data values within the range of the data given is called **interpolation** and is usually reasonably accurate. *Mastery lesson 3.4*
- Using a line of best fit to predict data values outside the range of the data given is called **extrapolation** and may not be accurate. *Mastery lesson 3.4*
- Individual points which are outside the overall pattern of a scatter diagram are called **outliers**. They can be removed from a data set provided a reason for their removal is given. *Mastery lesson 3.5*
- The line of best fit passes through the mean point, $(\bar{x}, \bar{y})$. *Extend 3*
- Means of time series data from several consecutive periods are called moving averages. *Extend 3*

4 Knowledge check

- It is often easier to write mixed numbers as improper fractions before doing a calculation. *Mastery lesson 4.1*
- You should divide by common factors before multiplying, if you can. *Mastery lesson 4.1*
- The **reciprocal** of the number n is $\frac{1}{n}$. You can also write this as n^{-1}. *Mastery lesson 4.1*
- To find the reciprocal of a fraction, swap the numerator and the denominator. For example, the reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$. *Mastery lesson 4.1*
- To find the reciprocal of a mixed number, first convert it into an improper fraction. *Mastery lesson 4.1*
- Sometimes both denominators must be changed to add fractions. *Mastery lesson 4.1*
- You can compare ratios by writing them as **unit ratios**. In a unit ratio, one of the numbers is 1. The other number may or may not be a whole number. *Mastery lesson 4.2*

- To share a quantity in a given ratio you could work out what fraction of the total amount each person receives, and then multiply each fraction by the total amount. Another method is to work out how much one part is worth, and then multiply by the number of parts each person receives. *Mastery lesson 4.2*
- When two quantities are in **direct proportion**, as one is multiplied by a number, n, so is the other. Their ratio also stays the same as they increase or decrease. *Mastery lesson 4.3*
- **Simple interest** is the interest calculated only on the original amount invested. It is the same each year. *Mastery lesson 4.4*
- You can calculate a percentage change using the formula

$$\text{percentage change} = \frac{\text{actual change}}{\text{original amount}} \times 100$$

Mastery lesson 4.4

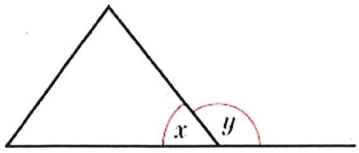
- $$\text{Percentage loss (or profit)} = \frac{\text{actual loss (or profit)}}{\text{original amount}} \times 100$$

You can use inverse operations to find the original amount after a percentage increase or decrease. *Mastery lesson 4.4*

- **Value Added Tax (VAT)** is charged at 20% on most goods and services. Domestic fuel bills have a lower VAT rate of 5%. On some things no VAT is charged. *Mastery lesson 4.4*
- **Depreciates** means loses value. *Mastery lesson 4.4*
- Your income means the amount of money you earn or are paid, and 'per annum' (abbreviated to p.a.) means each year. *Mastery lesson 4.4*
- When you are working out profits, remember to subtract any costs first. *Mastery lesson 4.4*
- All recurring decimals can be written as exact fractions. *Mastery lesson 4.5*
- If 1 decimal place recurs, multiply by 10.
  If 2 decimal places recur, multiply by 100.
  If 3 decimal places recur, multiply by 1000. *Mastery lesson 4.5*

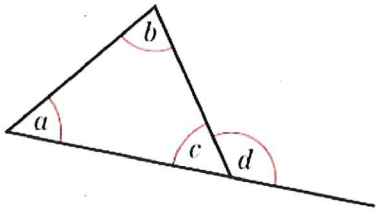
5 Knowledge check

- The angle marked x is called the **interior angle**. The angle marked y is called the **exterior angle**.



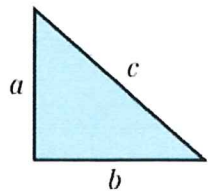
$x + y = 180°$ (angles on a straight line add up to 180°) ..... *Mastery lesson 5.1*

- For any polygon, interior angle + exterior angle = 180°. ..... *Mastery lesson 5.3*
- The exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices.



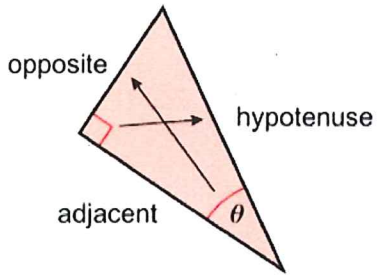
angle d = angle a + angle b ..... *Mastery lesson 5.1*

- The sum of the interior angles of a polygon with n sides = $(n - 2) \times 180°$. *Mastery lesson 5.2*
- The sum of the exterior angles of a polygon is always 360°. ..... *Mastery lesson 5.3*
- The exterior angle of a regular n-sided polygon is ..... *Mastery lesson 5.3*
- In a right-angled triangle the longest side is called the **hypotenuse** and is opposite the right angle. ..... *Mastery lesson 5.4*
- Pythagoras' theorem states that in a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

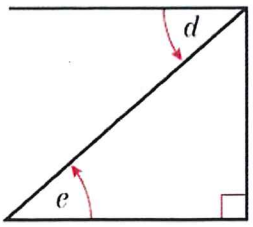


$c^2 = a^2 + b^2$ ..... Mastery lesson 5.4

- A triangle with sides a, b and c, where c is the longest side, is right-angled *only* if $c^2 = a^2 + b^2$. ..... Mastery lesson 5.4
- In a right-angled triangle, the side opposite the angle θ is called the **opposite**. The side next to the angle θ is called the **adjacent**. ..... Mastery lesson 5.6



- The **sine** of angle θ is the ratio of the opposite side to the hypotenuse, $\sin\theta = \frac{\text{opp}}{\text{hyp}}$. ..... Mastery lesson 5.6
- The **cosine** of angle θ is the ratio of the adjacent side to the hypotenuse, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$. ..... Mastery lesson 5.6
- The **tangent** of angle θ is the ratio of the opposite side to the adjacent side, $\tan\theta = \frac{\text{opp}}{\text{adj}}$. ..... Mastery lesson 5.6
- You can use $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$ on your calculator to find an angle when you know its sin, cos or tan. ..... Mastery lesson 5.7
- The **angle of elevation** (e) is the angle measured upwards from the horizontal. The **angle of depression** (d) is the angle measured downwards from the horizontal. ..... Mastery lesson 5.6



- The sine, cosine and tangent of some angles may be written exactly. . . *Mastery lesson 5.7*

	30°	45°	60°	0	90°
sin	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	0	1
cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	1	0
tan	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	0	

6 Knowledge check

- A **linear equation** generates a straight-line (linear) graph. ..... *Mastery lesson 6.1*
- The equation for a straight-line graph can be written as $y = mx + c$ where m is the gradient and c is the y-intercept. ..... *Mastery lesson 6.1*
- **Parallel lines** have the same gradient. ..... *Mastery lesson 6.1*
- To find the y-intercept of a graph, find the y-coordinate where $x = 0$.
  To find the x-intercept of a graph, find the x-coordinate where $y = 0$. ..... *Mastery lesson 6.1*
- To compare the gradients and y-intercepts of two straight lines, make sure their equations are in the form $y = mx + c$. ..... *Mastery lesson 6.1*
- A linear function has a graph that is a straight line. ..... *Mastery lesson 6.2*
- A **distance–time graph** represents a journey.
  - Straight lines mean constant speed
  - horizontal lines mean no movement
  - the gradient is the speed, since average speed = $\frac{\text{total distance}}{\text{total time}}$
  - Average speed = $\frac{\text{total distance}}{\text{total time}}$

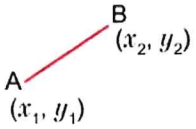
  Make sure your units match. ..... *Mastery lesson 6.3*
- The gradient of a straight-line graph is the rate of change. ..... *Mastery lesson 6.3*

- On a **velocity-time graph**
  - straight lines mean constant acceleration
  - horizontal lines mean no change in velocity (i.e. travelling at a constant velocity)
  - the gradient is the acceleration, since acceleration = $\frac{\text{change in velocity}}{\text{time}}$
  - the area under a velocity–time graph is the distance travelled.

- Graph axes do not have to start at zero.
  A zigzag line ⟋⟍ shows that values have been missed out. ..... *Mastery lesson 6.4*

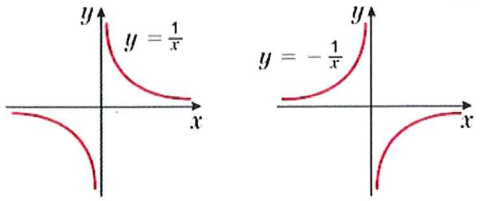
- When two quantities are in **direct proportion**
  - the graph is a straight line through the origin
  - when one variable is multiplied by n, so is the other. ..... *Mastery lesson 6.4*

- The coordinates of the **midpoint** of a line segment are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ .... *Mastery lesson 6.5*

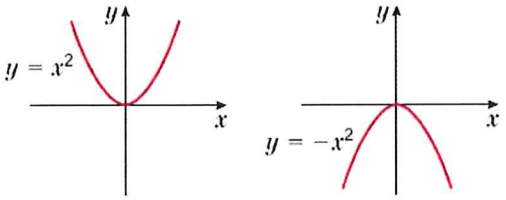


- When two lines are perpendicular, the product of the gradients is -1.
  When a graph has gradient m, a graph perpendicular to it has gradient $\frac{-1}{m}$. ... *Mastery lesson 6.5*

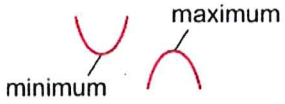
- **Reciprocal functions** are in the form $\frac{k}{x}$ where k is a number. ..... *Mastery lesson 6.7*



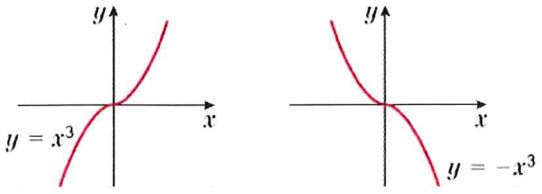
- A **quadratic equation** contains a term in x^2 but no higher power of x.
  The graph of a quadratic equation is a curved shape called a **parabola**. ..... *Mastery lesson 6.6*



- A quadratic graph has either a **minimum point** or a **maximum point** where the graph turns. ..... *Mastery lesson 6.6*



- A quadratic equation can have 0, 1 or 2 solutions. ..... *Mastery lesson 6.6*
- A **cubic function** contains a term in x^3 but no higher power of x.
  It can also have terms in x^2 and x and number terms. ..... *Mastery lesson 6.7*

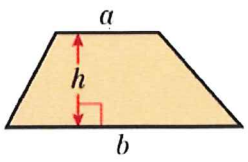


- A cubic function can have 1, 2 or 3 solutions. ..... *Mastery lesson 6.7*
- No correlation or weak correlation shows that there is *no* linear relationship between two quantities, because their graph is not close to a straight line.
  When the points follow a curve, there may be a non-linear relationship between the quantities. ..... *Mastery lesson 6.8*
- The equation of a circle with centre (0, 0) and radius r is $x^2 + y^2 = r^2$. ..... *Mastery lesson 6.8*

In this unit, which was easier:

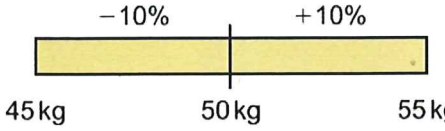
7 Knowledge check

- Area of a trapezium $= \frac{1}{2}(a + b)h$



Mastery lesson 7.2

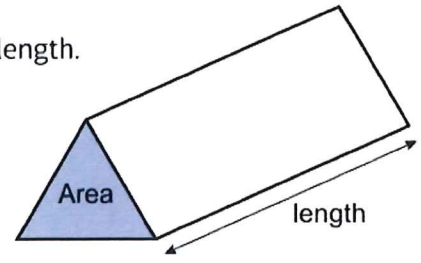
- To convert from cm^2 to mm^2, multiply by 100. To convert from mm^2 to cm^2, divide by 100. Mastery lesson 7.2
- 1 hectare (ha) is the area of a square 100 m by 100 m.
  1 ha = 100 m × 100 m = 10 000 m^2 Mastery lesson 7.2
- A 10% error interval means that a measurement could be up to 10% larger or smaller than the one given.



Mastery lesson 7.2

- Measurements rounded to the nearest unit could be up to half a unit smaller or larger than the rounded value. The possible values of x that round to 3.4 to 1 d.p. are $3.35 \leqslant x < 3.45$ Mastery lesson 7.2
- The upper bound is half a unit greater than the rounded measurement. The lower bound is half a unit less than the rounded measurement. Mastery lesson 7.2

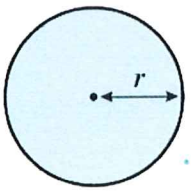
  $12.5 \leqslant x < 13.5$
  lower bound upper bound
- When giving the answer to a calculation to an appropriate degree of accuracy, round the upper and lower bounds by the same amount. If the upper and lower bound give the same value when rounded, then the answer is to an appropriate degree of accuracy. Mastery lesson 7.2
- **Volume** is measured in mm^3, cm^3 or m^3. Mastery lesson 7.3
- **Capacity** is measured in ml and litres.
- $1\,cm^3 = 1$ ml, $1000\,cm^3 = 1$ litre Mastery lesson 7.3
- The **surface area** of a 3D solid is the total area of all its faces. Mastery lesson 7.3
- A **prism** is a 3D solid that has the same cross-section all through its length. Mastery lesson 7.3
- Volume of a prism
  = area of cross-section × length.



Mastery lesson 7.3

- The **circumference** of a circle is its perimeter. For any circle
  circumference = π × diameter
  $C = \pi d$ or $C = 2\pi r$ Mastery lesson 7.4

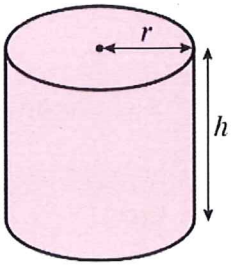
- The formula for the area, A, of a circle with radius r is $A = \pi r^2$



Mastery lesson 7.4

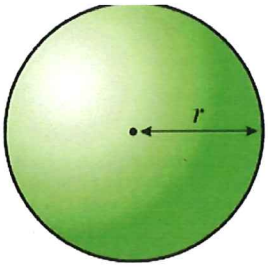
- The volume of a cylinder of radius r and height h is $V = \pi r^2 h$ ..... Mastery lesson 7.6

- The surface area of a cylinder of radius r and height h is $2\pi r^2 + 2\pi rh$



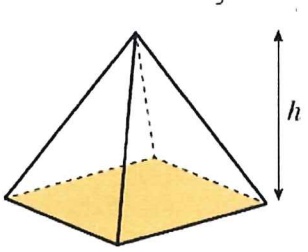
Mastery lesson 7.6

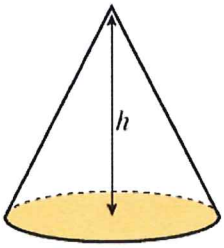
- For a sphere of radius r
  surface area = $4\pi r^2$
  volume = $\frac{4}{3}\pi r^3$



Mastery lesson 7.6

- Volume of pyramid = $\frac{1}{3}$ area of base × vertical height ..... Mastery lesson 7.7
- Volume of cone = $\frac{1}{3}$ area of base × vertical height = $\frac{1}{3}\pi r^2 h$ ..... Mastery lesson 7.7

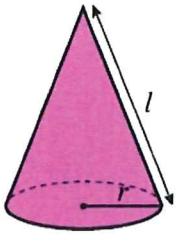




Mastery lesson 7.7

- Curved surface area of a cone = πrl, where r is the radius and l is the slant height. ..... Mastery lesson 7.7

- Total surface area of a cone = $\pi rl + \pi r^2$.



Mastery lesson 7.7

8 Knowledge check

- The **plan** is the view from above an object. The **front elevation** is the view of the front of the object. The **side elevation** is the view of the side of the object. *Mastery lesson 8.1*
- A **transformation** moves a shape to a different position. **Reflections**, **rotations**, **translations** and **enlargements** are all types of transformation. *Mastery lessons 8.2 and 8.3*
- An original shape is called an **object**. When the object is reflected, rotated, translated or enlarged, the resulting shape is called an **image**. *Mastery lessons 8.2 and 8.3*

- To describe a **rotation** you need to give the direction of turn (clockwise or anticlockwise), the angle of turn and the **centre of rotation**. *Mastery lesson 8.2*
- An **enlargement** is a transformation where all the side lengths of a shape are multiplied by the same **scale factor**. *Mastery lesson 8.3*
- To describe an enlargement you need to give the **centre of enlargement** and the scale factor. To find the centre of enlargement, join corresponding points of the object and the image. *Mastery lesson 8.3*
- To enlarge a shape by a fractional scale factor, multiply the distance from the centre to each point on the shape by the scale factor. *Mastery lesson 8.3*
- A negative scale factor takes the image to the opposite side of the centre of enlargement. *Mastery lesson 8.3*
- When a shape is enlarged the area increases by (scale factor)^2. *Mastery lesson 8.3*
- You can describe a translation using a **column vector**. The column vector for a translation 2 squares right and 3 squares down is $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$.

The top number in the column vector gives the movement parallel to the x-axis and the bottom number gives the movement parallel to the y-axis. *Mastery lesson 8.4*

- The **resultant vector** is the vector that moves the original shape to its final position after a number of translations or other transformations. *Mastery lesson 8.4*
- In reflections, rotations and translations, the object and the image are **congruent**, as the lengths of the sides and the angles do not change. *Mastery lesson 8.4*
- In an enlargement, the object and the image are **similar**. *Mastery lesson 8.4*
- A **bearing** is an angle in degrees, clockwise from north. A bearing is always written using three digits. *Mastery lesson 8.5*
- To **construct** means to draw accurately using a ruler and compasses. *Mastery lesson 8.6*
- A **perpendicular bisector** cuts a line in half at right angles. *Mastery lesson 8.6*
- The shortest distance from a point to a line is perpendicular to the line. *Mastery lesson 8.6*
- An **angle bisector** cuts an angle exactly in half. *Mastery lesson 8.7*

- A **locus** is the set of all points that obey a certain rule. Often the locus is a continuous path. *Mastery lesson 8.8*
- The locus of a point that moves so it is always a fixed distance from a fixed point is a circle. *Mastery lesson 8.8*
- Points equidistant from two points lie on the perpendicular bisector of the line joining the two points. *Mastery lesson 8.8*
- Points equidistant from two lines lie on the angle bisector. *Mastery lesson 8.8*

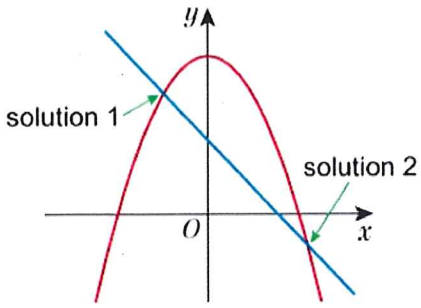
9 Knowledge check

- Solving a quadratic equation means finding values for the unknown that fit. *Mastery lesson 9.1*
- The **roots** of a quadratic function are its solutions when it is equal to zero. *Mastery lesson 9.1*
- You can solve equations of the form $ax^2 + bx + c = 0$ by fractorising. *Mastery lesson 9.2*
- You can use the quadratic formula to find the solutions to the **quadratic equation** $ax^2 + bx + c = 0$

  $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ *Mastery lesson 9.2*
- Expressions such as $(x + 2)^2$, $(x - 1)^2$ and $(x + \frac{1}{2})^2$ are called **perfect squares**. *Mastery lesson 9.3*
- $x^2 + bx + c$ can be written in the form $\left(x + \dfrac{b}{2}\right)^2 - \left(\dfrac{b}{2}\right)^2 + c$

  This is called **completing the square**. *Mastery lesson 9.3*
- $ax^2 + bx + c$ can be written as $a\left(x + \dfrac{b}{a}x\right)^2 + c$

  before completing the square for the expression inside the brackets. *Mastery lesson 9.3*
- When there are two unknowns, you need two equations to find their values. These are called **simultaneous equations**. *Mastery lesson 9.4*
- A pair of quadratic and linear simultaneous equations can have two possible solutions. *Mastery lesson 9.6*
- To find the coordinates where two graphs intersect, solve their equations simultaneously.



Mastery lesson 9.6

- You can show **inequalities** on a number line. An empty circle ○ shows that the value is not included. A filled circle ● shows that the value is included. An arrow ○⟶ shows that the solution continues towards infinity. *Mastery lesson 9.7*
- You can rearrange an inequality in the same way as you rearrange an equation. *Mastery lesson 9.7*
- You can write the solution to an inequality using **set notation**. *Mastery lesson 9.7*

$\{x : x > 2\}$

set of x such that

- When inequalities have a lower limit and an upper limit, solve the two sides separately. *Mastery lesson 9.7*
- When you multiply or divide an inequality by a negative number, reverse the inequality signs. *Mastery lesson 9.7*

10 Knowledge check

- A **sample space diagram** shows all the possible outcomes of two events. *Mastery lesson 10.1*
- Two events are **mutually exclusive** if they cannot happen at the same time. *Mastery lesson 10.2*
- When two events are mutually exclusive you can add their probabilities. The probabilities of an exhaustive set of mutually excusive events sum to 1. *Mastery lesson 10.2*
- For mutually exclusive events, P(not A) = 1 − P(A) *Mastery lesson 10.2*
- If there are m outcomes for one event and n outcomes for another event, the product rule states that the total number of outcomes for the two events is $m \times n$. *Mastery lesson 10.3*
- Expected number of outcomes = number of trials × probability. *Mastery lesson 10.3*
- Relative frequency = $\dfrac{\text{frequency}}{\text{total number of trials}}$ *Mastery lesson 10.3*
- As the number of experiments increases, the experimental probability gets closer and closer to the theorectical probability. *Mastery lesson 10.3*
- A **tree diagram** shows two or more events and their probabilities. *Mastery lesson 10.4*
- Two events are **independent** if one happening does not affect the probability of the other. *Mastery lesson 10.4*

- To find the probability of two independent events multiply their probabilities, P(A and B) = P(A) × P(B) ..... *Mastery lesson 10.4*
- The probability for a repeated independent event is the probability multiplied by itself, P(A and A) = P(A) × P(A), P(A and A and A) = P(A) × P(A) × P(A), etc. ..... *Mastery lesson 10.4*
- A **conditional probability** is when one outcome affects another outcome. ..... *Mastery lesson 10.5*
- P(A ∩ B) means the probability of the **intersection** of A and B. ..... *Mastery lesson 10.6*
- P(A ∪ B) means the probability of the **union** of A and B. ..... *Mastery lesson 10.6*
- P(A ∪ B) = P(A) + P(B) − P(A ∩ B) ..... *Mastery lesson 10.6*
- P(A ∩ B | B) means the probability of the intersection of A and B given B. ..... *Mastery lesson 10.6*

11 Knowledge check

- In **compound interest** the interest earned each year is added to money in the account and earns interest the next year. Most interest rates are compound interest rates. ..... *Mastery Section 11.1*
- Total interest = amount in the account at the end of the investment − amount invested ..... *Mastery Section 11.1*
- You can calculate an amount after n years' compound interest using the formula amount = initial amount × $\left(\frac{100 + \text{interest rate}}{100}\right)^n$ ..... *Mastery Section 11.1*
- Compound measures such as speed, density and pressure combine measures of two different quantities. ..... *Mastery Section 11.2 and 11.3*
- Speed can be measured in metres per second (m/s), kilometres per hour (km/h) or miles per hour (mph). ..... *Mastery Section 11.2*
- Average speed = $\frac{\text{distance}}{\text{time}}$ or $S = \frac{D}{T}$ ..... *Mastery Section 11.2*

- These are three kinematics formulae:
  $v = u + at$
  $s = ut + \frac{1}{2}at^2$
  $v^2 = u^2 + 2as$
  where a is constant acceleration, u is initial velocity, v is final velocity, s is displacement from the position when $t = 0$ and t is time taken. ... *Mastery Section 11.2*
- **Velocity** is speed in a given direction, possible units are m/s. ..... *Mastery Section 11.2*
- **Initial velocity** is speed in a given direction at the start of the motion. *Mastery Section 11.2*
- **Acceleration** is the rate of change of velocity, i.e. a measure of how the velocity changes with time, possible units are m/s^2. ..... *Mastery Section 11.2*
- Density is the **mass** of substance in g contained in a certain **volume** in cm^3 and is often measured in grams per cubic centimetre (g/cm^3).
  Density = $\frac{\text{mass}}{\text{volume}}$ or $D = \frac{M}{V}$ ..... *Mastery Section 11.3*
- Pressure is the **force** in newtons applied over an **area**, in cm^2 or m^2. It is usually measured in newtons (N) per square metre (N/m^2) or per square centimetre (N/cm^2).
  Pressure = $\frac{\text{force}}{\text{area}}$ or $P = \frac{F}{A}$ ..... *Mastery Section 11.3*

- When x and y are in direct proportion
  $y = kx$, where k is the gradient of the graph of y against x
  $\frac{y}{x} = k$, a constant ..... *Mastery Section 11.4*
- When x and y are in inverse proportion, y is proportional to $\frac{1}{x}$.
  As one doubles (×2) the other halves (÷2). ..... *Mastery Section 11.4*
- When x and y are in inverse proportion then
  $x \times y$ = a constant
  $xy = k$, so $y = \frac{k}{x}$ ..... *Mastery Section 11.4*

12 Knowledge check

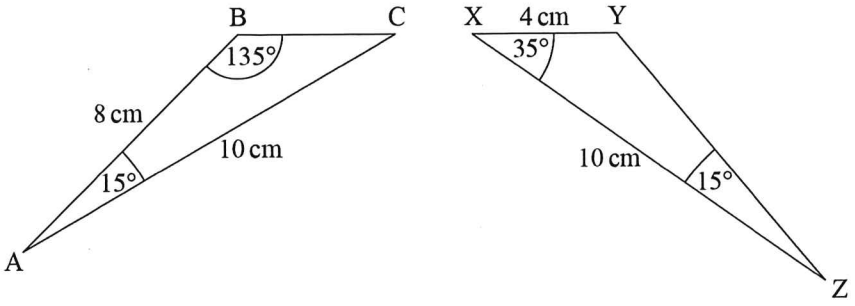
- Congruent triangles have exactly the same size and shape. Their angles are the same and corresponding sides are the same length. *Mastery lesson 12.1*
- Two triangles are congruent when one of these conditions of congruence is true.
  SSS (all three sides equal)
  SAS (two sides and the included angle are equal)
  AAS (two angles and a corresponding side are equal)
  RHS (right angle, hypotenuse and one other side are equal) *Mastery lesson 12.1*
- You can use congruence to solve problems and prove that shapes are the same. *Mastery lesson 12.2*
- To prove something, you write a series of logical statements that show the statement is true. Each statement must be supported by a mathematical reason. *Mastery lesson 12.2*

- Shapes are similar when one shape is an enlargement of the other. Corresponding angles are equal and corresponding sides are all in the same ratio. *Mastery lesson 12.3*
- When a shape is enlarged by linear scale factor k, the area of the shape is enlarged by scale factor k^2. *Mastery lesson 12.4*
- When a shape is enlarged by linear scale factor k, the volume is enlarged by scale factor k^3. *Mastery lesson 12.5*
- When the linear scale factor is k:
  Lengths are multiplied by k
  Area is multiplied by k^2
  Volume is multiplied by k^3 *Mastery lesson 12.5*

NAME

1 ABC and XYZ are triangles.

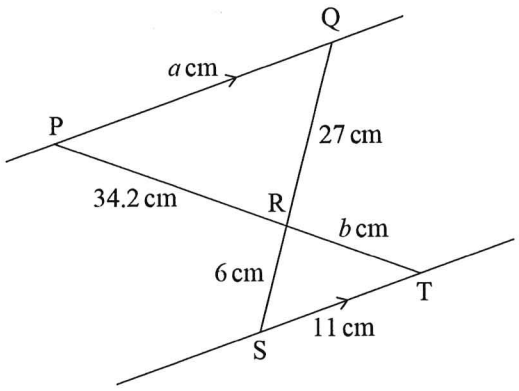
Show that the triangles are **not** similar.



(4 marks)

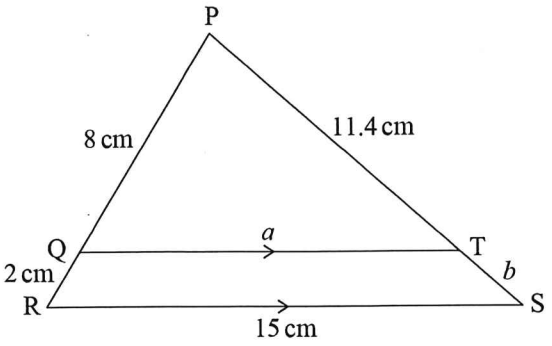
2 Work out the lengths a and b.

Explain all the steps in your working.



(4 marks)

3 QT is parallel to RS.



Work out

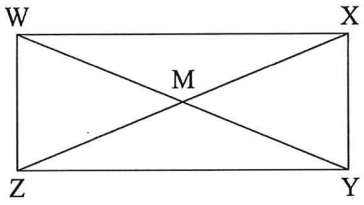
a the length of QT

(2 marks)

b the length of ST.

(2 marks)

4 WXYZ is a rectangle. WY and XZ are the diagonals of the rectangle, which cross at point M.



a Prove that both diagonals divide the rectangle into two congruent triangles.

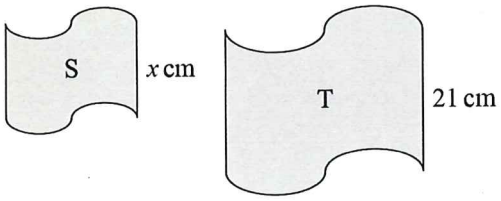
(4 marks)

b Prove that M is the midpoint of WY and XZ.

(2 marks)

5 Shapes S and T are similar.

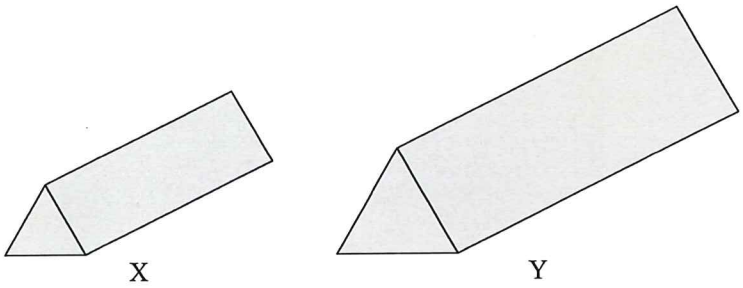
The area of S is 22cm^2. The area of T is $269.5\,\text{cm}^2$.



Find the length of side x.

(3 marks)

6 X and Y are mathematically similar chocolate boxes.



X has a volume of 130cm^3 and a surface area of 202cm^2.

The volume of Y is 638.69cm^3.

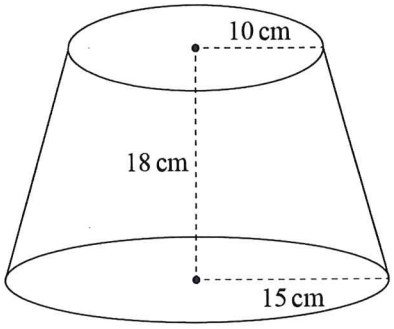
What is the surface area of Y?

(4 marks)

7 Work out the volume of the frustum.

Give your answer correct to 3 significant figures.

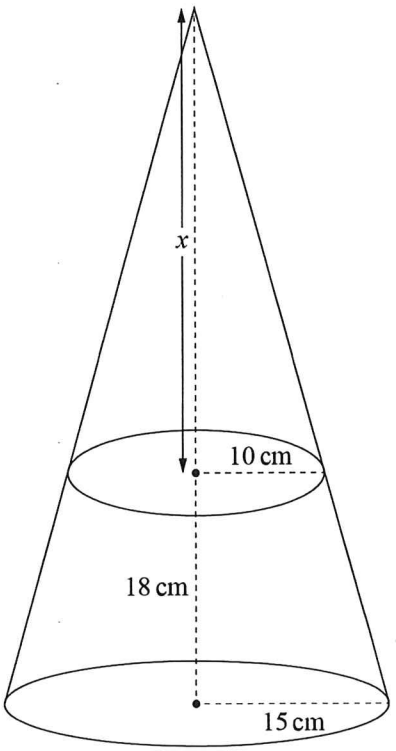
Volume of a cone $= \frac{1}{3}\pi r^2 h$



(5 marks)

Overall mark	**/30**

Q	Answer	Mark	Comment
1	angle BCA = 30°	M1	
	angle XYZ = 130°	M1	
	Similar triangles must have identical angles, so the two triangles are not similar.	A2	A1 for 'similar triangles must have identical angles', and A1 for 'so the two triangles are not similar'.
2	Angle QPR = angle RTS alternate angles		
	Angle PQR = angle RST alternate angles		
	Angle PRQ = angle SRT opposite angles		
	So PQR and RST are similar triangles.	M1	
	Scale factor = $\frac{27}{6} = 4.5$	M1	
	$a = 49.5\,\text{cm}$	A1	
	$b = 7.6\,\text{cm}$	A1	
3a	Scale factor PRS to PQT = $\frac{4}{5}$	M1	
	$a = 12\,\text{cm}$	A1	
3b	$b = 14.25 - 11.4$	M1	
	$= 2.85\,\text{cm}$	A1	
4a	Opposite sides are parallel and equal in a rectangle, so WX = YZ and WZ = XY.	A1	
	All vertices in WXYZ are 90°, so triangle XWZ, triangle XYZ, triangle WXY and triangle WYZ are congruent (SAS).	A1	
	Therefore angle XWY = angle WYZ = angle XZY = angle WXZ and angle ZWY = angle ZXY = angle WZX = angle WYX (alternate angles).	A1	
	Therefore triangles WMX and ZMY are congruent and triangles WMZ and XMY are congruent.	A1	

Q	Answer	Mark	Comment
4b	Triangles WMX and ZMY are congruent so their sides must be the same length. The base angles are equal so they are isosceles triangles so WM = XM = ZM = YM.	A1	
	Therefore M is the midpoint of both WY and XZ.	A1	
5	Area scale factor of S to T = $\frac{269.5}{22}$ = 12.25	M1	
	Linear scale factor = $\sqrt{12.25}$ = 3.5	M1	
	x = 6 cm	A1	
6	Volume scale factor of X to Y = $\frac{638.69}{130}$ = 4.913	M1	
	Linear scale factor = $\sqrt[3]{4.913}$ = 1.7	M1	
	Area scale factor = 1.7^2 = 2.89	M1	
	Surface area of Y = 583.78 cm^2	A1	
7			

Q	Answer	Mark	Comment
	$\frac{x}{10} = \frac{x+18}{15}$	M1	
	$15x = 10(x + 18)$	M1	
	$x = 36$ cm		
	Volume of large cone = 4050π	M1	
	Volume of small cone = 1200π	M1	
	Volume of frustum = volume of large cone – volume of small cone = 8950cm^3 (3 s.f.)	A1	

Progression Step Boundaries

Mark boundary	Step
0	U
2	7th
4	8th
7	9th
11	10th
16	11th
22	12th

Question	1	2	3	4	5	6	7
Objective	Prove triangles are not similar.	Find missing lengths in similar triangles at parallel lines.	Work out missing lengths in similar triangles, one inside the other with one pair of parallel sides.	Prove that a rectangle is divided into two congruent triangles along its diagonal, and that the diagonals bisect.	Find missing lengths in similar shapes, from given areas.	Work out surface area of similar 3D shapes, from given volume information.	Work out the volume of a frustum.
Step	8th	9th	9th	12th	11th	12th	12th
Marks	4	4	4	6	3	4	5